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AN ECONOMETRIC ANALYSIS OF THE INTERTEMPORAL
GENERAL EQUILIBRIUM APPROACH TO EXCHANGE
RATE AND CURRENT ACCOUNT DETERMINATION

by

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Abstract

This paper attempts the econometric implementation of a monetary, intertemporal general-equilibrium model of exchange rate and current account determination in a small open economy. The reduced form of the model comprises a statement of the permanent income hypothesis of consumption and the rational expectations extension of the monetary approach to exchange rate determination. The model is estimated using full-information-maximum-likelihood methods and quarterly data over the recent flexible exchange rate period on the UK and US economies. The empirical findings suggest that the model shows considerable promise in explaining the exchange rate and consumption side of current account behavior. Some results are, however, suggestive of omitted considerations and extensions of the model for future research are indicated.

This paper pursues the econometric implementation of an intertemporal general-equilibrium (GE) model of exchange rate and current account determination. The model is, essentially, based on the type of framework used recently in Obstfeld (1981) and Greenwood (1983). In this model agents are viewed as facing perfect goods and asset markets and choosing consumption and investment plans so as to maximize the utility from consumption and real money balances over an infinite time horizon subject to an intertemporal budget constraint. Reflecting the intertemporal nature of the model, it follows that current and expected future real opportunities and monetary policies play a key role in the money demand decision that governs the evolution of the exchange rate, while current and expected future real opportunities influence the consumption-savings decision¹ that determines the current account. Accordingly, the model highlights that the relationship between exchange rate and current account movements depends crucially on the type of shock to the economy and on whether it is perceived as a transitory or permanent disturbance. This feature is elucidated in Sachs (1981) and Greenwood (1983).

The undertaking in this study is especially of interest in view of the following considerations. Firstly, there have been remarkably few attempts to estimate and test intertemporal GE models of exchange rate and current account behavior, despite the growing preponderance of such models in the theoretical international finance literature [see e.g., the above references, Helpman and Razin (1982) and Stockman and Svensson (1987)]. Indeed, to my knowledge, there are only two innovative empirical studies falling within this area: Hercowitz (1986) investigates whether the capital account can be explained by a real, intertemporal GE small-open economy model using data on the Israeli economy. Although, the results indicate mixed support for the model, they are encouraging.

Hodrick (1987) investigates whether the exchange rate can be explained by a monetary, intertemporal GE two-country model using data on the US, UK, West German and Japanese economies. The results are not supportive of the model. However, the latter study employs a restrictive specification of the demand for money--one with a unitary real income velocity and the estimation is preliminary in nature. The present study pursues the estimation and testing of a monetary, intertemporal GE small-open economy model of the joint determination of the exchange rate and current account, in which income velocity is not restricted to unity and the econometric techniques are rigorously justified. Secondly, most of the existing (other) empirical exchange rate models are not supported by the data. More specifically, exchange rate models in the tradition of the monetary (both flexible and sticky-price versions) and portfolio balance approaches typically reveal insignificant and/or incorrectly signed coefficients when estimated [see Backus (1984) for a comprehensive study in this regard]. Furthermore, the Meese and Rogoff (1983) forecasting analysis and the Backus (1984) regression analysis of these models show that the random walk model of exchange rates typically provides the best approximation to observed exchange rate dynamics. These studies, leave open the question as to how we can improve on our explanations of exchange rate behavior and establish the random walk model as a benchmark against which to evaluate the tracking and forecasting performance of structural exchange rate models. The present study attempts to shed some light on this question by examining the consistency of an intertemporal GE model with the data.

An important advantage of econometrically implementing an intertemporal GE model is that it permits the estimation of the 'deep structural' parameters of taste as well as the parameters of the exogenous driving processes in the

economy, rather than the parameters of consumption/savings and money demand functions. As Lucas (1976) has shown, estimates of these latter parameters may confound those of the 'deep structural' parameters with estimates of the parameters of the exogenous driving processes and, accordingly, may not be stable over time. The econometric model in this paper is, therefore, more robust with respect to this kind of instability than are standard econometric models of consumption and money demand. The reduced form of the model essentially comprises a statement of the permanent income hypothesis of consumption and the rational expectations extension of the monetary approach to exchange rate determination. The model is operationalized using the results of Hansen and Sargent (1980) and Flavin (1981) for operationalizing infinite-discounted sums of expectational terms and Box-Jenkins time-series techniques to characterize the exogenous driving processes. The model is then estimated using full-information-maximum-likelihood methods and quarterly data over the recent flexible exchange rate period on the UK and US economies, where the UK is taken as the small-open economy.

The remainder of the paper is organized as follows: Section 1 sets out the model which serves as the framework for the empirical analysis; Section 2 contains some preliminary implementation considerations; Section 3 discusses the empirical findings and Section 4 concludes the paper.

I. The Model

The model describes a small open economy that is inhabited by identical infinitely-lived agents and that operates under a flexible exchange rate. Agents have perfect foresight and maximize utility from consumption and real money balances over an infinite time horizon subject to an intertemporal budget

constraint. One good exists in this world, so that domestic and foreign goods are perfect substitutes; therefore, abstracting from transportation costs and trade impediments, purchasing power parity holds. Agents are (exogenously) endowed with units of the good each period. The domestic government taxes agents and issues a currency which is held only by nationals. No foreign money is held by domestic residents. Also, domestic residents (agents and government) can freely participate on an international bond market at a constant real interest rate and interest rate parity (real and uncovered nominal) holds.

Some comments on this proposed structure are in order:

First, it is clear that the analytical framework is a simple one. Some may fault, in particular, the assumptions of purchasing power parity, uncovered interest rate parity and real interest rate constancy, in view of empirical evidence to the contrary [Frenkel (1981), Cumby and Obstfeld (1982), Mishkin (1984)]. However, maintaining those assumptions--shared as they are with most existing empirical exchange rate models--allows us to focus on the contribution of the empirical-intertemporal GE framework above existing empirical models in explaining exchange rate and current account behavior.

Second, with respect to the assumption of perfect foresight--this raises the issue of using a deterministic framework as a basis for empirical work. Clearly, in general, a stochastic framework is preferable but is not used here due to difficulties in its econometric implementation. In particular, while it is possible (using parameter restrictions) to derive a closed-form solution for the stochastic model--one where the endogenous variables depend only on exogenous and pre-determined variables--it is highly nonlinear in variables and parameters. The likelihood function for the latter model is not well behaved, giving rise to extremely unstable parameter estimates.² On the other hand, the closed-form

solution for the deterministic model does not require parameter restrictions, is linear in the variables and does not have the estimation problems evident for the stochastic model.

Third, the model explains only the consumption (or alternatively savings) side of current-account determination, leaving the investment side exogenous. Most intertemporal GE models of the current account have in fact focussed exclusively on the savings-side of current-account determination (see e.g., Greenwood [1983], Obstfeld [1981], Svensson and Razin [1983], Hercowitz [1986], and Helpman and Razin [1982]). Studies which explain both the savings and investment side of the current account in an infinitely-lived representative agent framework include Brock (1988), Lipton and Sachs (1983), Obstfeld (1986) and Stockman and Svensson (1987). Although the former two incorporate adjustment costs, all of the studies contain simple formulations of investment determination--particularly in assuming that the installation of capital takes one period. Including a more realistic explanation of investment behavior--perhaps along the time-to-build lines of Kydland and Prescott (1982) --in an intertemporal GE model of the current account constitutes an ambitious task for future theoretical and empirical research. The importance of such an undertaking is underscored by the findings in Sachs (1981), viz: for the OECD countries, shifts in investments rates have typically dominated savings rate movements in explaining current account patterns. This task, however, lies outside the scope of the present study.

The fourth issue concerns modeling money. Currently, money is popularly introduced into intertemporal GE models in one of the following ways: (i) by entering money as an argument in agents' utility functions; (ii) by entering money into liquidity costs which appear in agents' budget constraints; (iii) by

imposing cash-in-advance (CIA) constraints on agents' goods purchases and (iv) by assuming money is the sole means of effecting intergenerational transfers i.e., the overlapping-generations (OLG) model of money. For the purpose of the present study, a money-in-the-utility function model turns out to be the most convenient and/or interesting alternative. More specifically: Motivating money via a liquidity cost specification--as in Greenwood (1983)--leads to a similar framework but requires a data series on liquidity costs for the empirical implementation. Motivating money via a CIA specification--as in Lucas (1982) and Helpman and Razin (1982)--also leads to a similar framework but gives rise to a more restrictive specification of the demand for money since the equilibrium of this model is characterized by a binding CIA constraint and thus a unitary income velocity of money. Generalizing the CIA model to admit the possibility of a non-binding CIA constraint and thus to allow for a variable income velocity--as in Lucas (1984), Svensson (1985) and Stockman and Svensson (1987)--gives rise to a model which, as of the present time, is difficult to solve explicitly.³ It is also interesting to note here that Feenstra (1986) shows that a CIA model may be viewed as a special case of a money-in-the-utility function model; namely, one characterized by a zero elasticity of substitution across consumption and real money balances. Accordingly, it may be argued that the money-in-the-utility function model used in the present study is a more general model than a CIA model. Finally, note that the OLG model of money has been criticized for neglecting the transactions demand for money--probably its main *raison d'être*--and is unlikely to serve as a useful basis for the econometric analysis of short-run time series data.

The model may be more formally described as follows. Consider an environment in which the representative agent has preferences, defined over consumption and real money balance sequences, given by:

$$(1) \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{c_t^{(1-\sigma)}}{(1-\sigma)} + \frac{(M_t^d/P_t)^{(1-\delta)}}{(1-\delta)} \right]$$

$$0 < \beta < 1 \quad \sigma, \delta > 0$$

where: c_t , M_t^d and P_t respectively denote time- t real consumption, desired nominal money holdings and the nominal price level; β is the subjective discount factor, assumed to be a positive fraction reflecting the assumption of a positive rate of time preference; σ , δ are preferences parameters, assumed to be positive on the assumption of a concave momentary utility function with respect to consumption and real money balances.⁴ The agent is assumed to maximize (1) by choosing c_t , M_t^d and b_t^p subject to the budget constraint:

$$(2) \quad b_t^p = y_t - \tau_t - c_t - \frac{(M_t^d - M_{t-1}^d)}{P_t} + (1 + r^*)b_{t-1}^p$$

$$b_0^p = 0$$

where: b_t^p , y_t and τ_t respectively denote real time- t one-period bond purchases, endowment income and lump-sum tax and r^* is the constant foreign real interest rate. Equation (2) sets the sum of the real values of the agent's current consumption, bond and money choices equal to the sum of the real values of the agent's current after-tax endowment income, last period's money holdings and the gross real return from last period's bond purchases.

The first-order conditions consist of (2) and:

$$(3) \quad c_t^{-\sigma} = \beta(1 + r^*)c_{t+1}^{-\sigma}$$

$$(4) \quad (M_t^d/P_t)^{-\delta} = c_t^{-\sigma} - \beta(P_t/P_{t+1})c_{t+1}^{-\sigma}$$

Equation (3) is the Euler equation governing the agent's optimal choice of bonds. By acquiring an additional bond at time t the agent foregoes consumption - the marginal cost of which is measured by the time- t marginal utility of consumption, $c_t^{-\sigma}$. The gross real return from the bond, $(1+r^*)$, permits increased consumption at time $(t+1)$ --the discounted marginal utility of which is given by $\beta(1+r^*)c_{t+1}^{-\sigma}$. Accordingly, (3) sets the marginal cost equal to the discounted marginal benefit of acquiring an additional bond. Notice from equation (3) that $(1/\sigma)$ is the elasticity of intertemporal substitution i.e. it measures the responsiveness of the percentage change in consumption to changes in (the logarithm of) the gross real interest rate.⁵ Equation (4) is the Euler equation governing the agent's optimal choice of money holdings. By acquiring an additional nominal money unit at time t the agent foregoes $1/P_t$ units of consumption - the marginal cost of which is measured by $c_t^{-\sigma}(1/P_t)$. The extra nominal money unit directly yields utility at time t --in particular, the time- t marginal utility of this money unit is given by $[(M_t^d/P_t)^{-\delta}1/P_t]$. Furthermore, the additional nominal money unit is worth $1/P_{t+1}$ units of goods at time $(t+1)$ --the discounted marginal utility of which is measured by $[\beta(1/P_{t+1})c_{t+1}^{-\sigma}]$. Therefore, (4) is seen to set the marginal cost equal to the total marginal benefit of acquiring an additional nominal money unit.

The government's budget constraint is:

$$(5) \quad b_t^g + g_t = \tau_t + (1 + r^*)b_{t-1}^g + (M_t - M_{t-1})/P_t, \quad b_0^g = 0$$

where: b_t^g , g_t and M_t denote, respectively, time- t government one-period real bond and goods purchases and the nominal money supply. Equation (5) sets the sum of the real values of the government's current bond and goods purchases equal

to the sum of the real values of current tax revenue, the gross real return from last period's government bond purchases and current seignorage revenue. From (5) it is clear that money enters the economy, modeled here, through government purchases, lump-sum transfers or open-market operations.

Assuming $M_t^d = M_t$ for all t and substituting (5) into (2), the economy-wide budget constraint is obtained:

$$(6) \quad b_t = y_t - c_t - g_t + (1 + r^*)b_{t-1}, \quad b_t = b_t^p + b_t^g, \quad b_0 = 0$$

i.e., the sum of the current real values of the economy's total goods and bond purchases equals the sum of current real endowment income and the gross real return from the economy's last-period bond holdings. Summing the one-period budget constraints reveals that the economy faces an intertemporal budget constraint of the form:

$$(7) \quad \sum_{s=0}^T \frac{c_{t+s}}{(1+r^*)^s} = \sum_{s=0}^T \frac{(y_{t+s} - g_{t+s})}{(1+r^*)^s} - \frac{b_{t+T}}{(1+r^*)^T} + (1+r^*)b_{t-1}$$

As $T \rightarrow \infty$, the transversality condition:

$$\lim_{T \rightarrow \infty} \frac{b_{t+T}}{(1+r^*)^T} = 0 \text{ is imposed on (7) to give:}$$

$$(8) \quad \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r^*)^s} = \sum_{s=0}^{\infty} \frac{(y_{t+s} - g_{t+s})}{(1+r^*)^s} + (1+r^*)b_{t-1} \equiv W_t$$

where W_t denotes wealth as of time t . Equation (8) sets the present discounted value of real consumption equal to the present discounted value of real endowment income less government expenditures plus the gross real return from the economy's last-period bond purchases. It is important to impose the transversality condition in the present setting of perfect capital mobility, since it rules out

the possibility that the economy can attain unbounded consumption by borrowing arbitrarily large sums on the world capital market and meeting all interest payments through further borrowing.

The general equilibrium solution of the model involves the simultaneous solution of equations (3), (4) and (8)--with $M_t^d = M_t$ in equation (4). These equations show that the Ricardian Equivalence Theorem holds in this model i.e. for any given time stream of government expenditures, the time path of real consumption is unaffected by alternative financing schemes for government purchases involving taxation and bond issue. This feature follows from the fact that in this model the time horizon of private agents and government coincide, taxation is lump-sum in nature and the private sector recognizes the finance constraint of the government. Also notice that government expenditures do not affect the real interest rate--as this is an exogenous given, reflecting the small open economy assumption. In a more general model which admitted a time-varying and endogenous real interest rate, government expenditures would have real interest rate effects along the lines modeled in Barro (1981). If, in addition, the assumptions underlying the Ricardian Equivalence Theorem were relaxed, then alternative financing schemes involving taxation and bond issue would have implications not only for consumption but also the real interest rate. Finally note from equations (3), (4) and (8) that money is both neutral and superneutral in this model. These neutrality results would breakdown if the preference specification was generalized to admitting the dependence of the marginal utility of consumption on real money balances.

In order to obtain the general equilibrium solution of the model, first iterate on (3) to obtain:

$$(9) \quad c_{t+s} = [\beta(1 + r^*)]^{s/\sigma} c_t$$

Substituting (9) into (8) and rearranging gives:⁶

$$(10) \quad c_t = mW_t, \quad m \equiv 1 - (1 + r^*)^{-1} [\beta(1 + r^*)]^{1/\sigma}$$

i.e., consumption is proportional to wealth. This is a statement of the permanent income hypothesis of consumption--which is, perhaps, clearer to see when one notes that wealth and permanent income bear a proportionate relationship to one another at a constant real interest rate, viz: $W_t = [(1 + r^*)/r^*] y_t^p$, where y_t^p denotes permanent (disposable) real income. This equation reflects the forward-looking intertemporal nature of the model. Recalling that the current account is, by definition, income less absorption, it follows from (10) that the current account will exhibit little (a significant) response to permanent (temporary) changes in output and government expenditures. More specifically, as shown in Hercowitz (1986), the more persistent is the disturbance to output and/or government expenditures, the greater is its impact on wealth and hence on current consumption--accordingly, the effect of the disturbance on the current account is inversely related to its persistence.

The next set of steps used in obtaining the general equilibrium solution of the model, essentially, involves the imposition of the money market equilibrium condition: $M_t^d = M_t$ and working with equations (3), (4) and:

$$(11) \quad i_t = i_t^* + \log e_{t+1} - \log e_t$$

$$(12) \quad P_t = P_t^* e_t$$

$$(13) \quad i_t^* = r^* + \log P_{t+1}^* - \log P_t^*$$

where: $i_t(i_t^*)$ denotes the domestic (foreign) net nominal interest rate between time t and time $(t+1)$; e_t and P_t^* denote, respectively, the time- t domestic currency price of one unit of foreign currency and the foreign price level and

log denotes the natural logarithm. Equation (11) is a statement of the uncovered interest rate parity condition, viz: the nominal interest rate differential across the domestic and foreign economies reflects the depreciation of the exchange rate over the corresponding investment period. Equation (12) states the purchasing power parity condition i.e., price levels are equal across countries when expressed in the same currency unit. Equation (13) expresses the Fisher relationship for the foreign nominal interest rate, viz: the difference between the foreign nominal and real interest rate reflects foreign price inflation over the associated investment period. The following relationship for the exchange rate emerges:

$$\begin{aligned}
 (14) \log e_t = & \frac{\delta i_0}{1 + \delta i_0} \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \log M_{t+j} \\
 & + \frac{1}{(1 + \delta i_0)} \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \log P_{t+j+1}^* \\
 & - \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \log P_{t+j}^* \\
 & - (\sigma/\delta) \log c_t + f_0
 \end{aligned}$$

where:

$$f_0 = s_0 \frac{(1 + \delta i_0)}{\delta i_0} - \frac{\sigma}{(\delta^2 i_0)} \log [\beta(1 + r^*)]^{1/\sigma}$$

i_0 denotes the mean of i_t and s_0 is a constant term. The complete details on the derivation of equation (14) are provided in Appendix 2. This equation shows that the current value of the exchange rate depends on the current and infinite-future values of the domestic money supply and foreign price level as well as on current consumption. As such, it is, essentially, a statement of the rational expectations extension of the monetary approach to exchange rate determination

(once the future values of the exogenous variables are replaced by their expected values, conditioned on all available information--see e.g. Bilson (1978) and Finn (1986)). This is, perhaps, all the more evident if (10) and the definition of permanent income are used to replace current consumption in (14) by permanent income and the parameter $(1/\delta i_0)$ is interpreted as the interest rate semi-elasticity of the demand for money.⁷ Equation (14) thus, also, reflects the forward-looking intertemporal nature of the model. The response of the exchange rate to temporary and permanent disturbances in the domestic money supply and foreign price level depends crucially on the magnitude of $(1/\delta i_0)$. High (low) values of this parameter imply that much (little) weight is attached to future values of these variables. Thus, in the former case the exchange rate will exhibit little response to temporary disturbances and a significant response to permanent disturbances. On the other hand, in the latter case the exchange rate will exhibit a similar response to both types of disturbances.

II. Implementation Considerations

II.A. Sample

The sample period for the empirical analysis is the second quarter of 1974 through to the last quarter of 1986 (after allowing for the lagged variables which will be indicated below). The UK (US) is assumed to be the domestic (foreign) economy. Thus, the small open economy assumptions noted above are expected to be satisfied. Quarterly data on the UK - US exchange rate, the UK money supply and real consumption and the US price level are employed. A complete description of the data and its sources is provided in Appendix 1.

II.B. The Consumption Function

For estimation purposes the consumption function specified in equation (3) is chosen rather than that specified in equation (10) since the former does not include wealth. More precisely, equation (3) is lagged one period and rearranged to give:⁸

$$(15) \quad c_t = [\beta(1 + r^*)]^{1/\sigma} c_{t-1}$$

II.C. The Exchange Rate Equation

The exchange rate equation [equation (14)] is operationalized by replacing the future values of the exogenous variables by their expected values, conditioned on current information and by obtaining observable expressions for the latter. This involved the following work.

First, an unconstrained vector autoregression was estimated, comprising the two exogenous variables: $\log M_t$ and $\log P_t^*$.⁹ Three lags were employed on each variable and since the joint estimation revealed negligible correlation across the equations' residuals, each equation was subsequently estimated using OLS. A sequence of F-tests suggested that neither exogenous variable Granger-causes the other. This suggests that, for forecasting purposes, univariate time series models of the exogenous variables are adequate.

Second, in order to obtain a parsimonious representation of the stochastic processes for the exogenous variables, the Box-Jenkins three-step, univariate, time-series analysis of identification, estimation and diagnostics was carried out. A prerequisite for this analysis is that the variables to be modeled are stationary. First differencing was required to achieve this. The following results were obtained. The identification step of the analysis proved to be

ambiguous. The estimation and diagnostic steps suggested that $\Delta \log M_t$ is adequately captured by an AR(2) process while $\Delta \log P_t^*$ seems to follow an AR(3) process¹⁰ i.e.

$$(16) \quad \alpha_1(L) \Delta \log M_t = v_{1t}, \quad \alpha_1(L) \equiv 1 - \alpha_{11}L - \alpha_{12}L^2$$

$$(17) \quad \alpha_2(L) \Delta \log P_t^* = v_{2t}, \quad \alpha_2(L) \equiv 1 - \alpha_{21}L - \alpha_{22}L^2 - \alpha_{23}L^3$$

where: α_{ij} is a parameter ($i = 1, 2$ $j = 1, 2, 3$); L (Δ) is the lag (first difference) operator and v_{it} is a white noise disturbance term ($i = 1, 2$). The results of the Box-Jenkins estimation of (16) and (17) are reported in Table 1. Also reported are the individual autocorrelations of the residuals, the Box-Pierce Q statistic and the critical value of the χ^2 for the Box-Pierce test. This test suggests that the residuals have been reduced to white noise. A further diagnostic check was undertaken by comparing the autocorrelation functions of the actual and simulated time series. This also suggested that the specifications in (16) and (17) are adequate. [See Pindyck and Rubinfeld (1976) for further details.]

Third, equations (16) and (17) are used to generate observable expressions for the infinite discounted sum of the first differences of the expectational terms in:

$$\begin{aligned}
(14') \quad \Delta \log e_t &= \frac{\delta i_0}{(1 + \delta i_0)} \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \Delta E_t \log M_{t+j} \\
&+ \frac{1}{(1 + \delta i_0)} \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \Delta E_t \log P_{t+j+1}^* \\
&- \sum_{j=0}^{\infty} \left(\frac{1}{1 + \delta i_0} \right)^j \Delta E_t \log P_{t+j}^* - \sigma/\delta \Delta \log c_t
\end{aligned}$$

where: E_t is the expectations operator conditioned on time- t information. Equation (14') is obtained from equation (14) by: replacing the future values of variables by their conditional expectations and first differencing. More specifically, equation (14') may now be operationalized using the results:

$$\begin{aligned}
(18) \quad \sum_{j=0}^{\infty} \phi^j \Delta E_t Z_{t+j} &\equiv \sum_{j=0}^{\infty} \phi^j E_t \Delta Z_{t+j} + \sum_{j=0}^{\infty} \phi^j (E_t - E_{t-1}) Z_{t+j-1} \\
&= \alpha(\phi)^{-1} \left[1 + \sum_{j=1}^{q-1} \left(\sum_{k=j+1}^q \phi^{k-j} \alpha_k \right) L^j \right] \Delta Z_t \\
&+ \phi(1-\phi)^{-1} \alpha(\phi)^{-1} \alpha(L) \Delta Z_t
\end{aligned}$$

and, by extension,

$$\begin{aligned}
(19) \quad \sum_{j=0}^{\infty} \phi^j \Delta E_t Z_{t+j+1} &\equiv \sum_{j=0}^{\infty} \phi^j E_t \Delta Z_{t+j+1} + \sum_{j=0}^{\infty} \phi^j (E_t - E_{t-1}) Z_{t+j} \\
&= \alpha(\phi)^{-1} \left[1 + \sum_{j=1}^{q-1} \left(\sum_{k=j+1}^q \phi^{k-j} \alpha_k \right) L^j \right] \left(\alpha_1 \Delta Z_t + \dots + \alpha_q \Delta Z_{t+1-q} \right) \\
&+ (1-\phi)^{-1} \alpha(\phi)^{-1} \alpha(L) \Delta Z_t
\end{aligned}$$

where: the covariance-stationary process, ΔZ_t , satisfies:

$$(20) \quad \alpha(L) \Delta Z_t = v_t, \quad \alpha(L) = 1 - \alpha_1 L - \dots - \alpha_q L^q$$

and where: ϕ and α_i are parameters with $0 < \phi < 1$ (and $i = 1, 2, \dots, q$) and v_t is a white noise disturbance term. The result in (18) is based on Hansen and Sargent's (1980) and Flavin's (1981) work in operationalizing infinite-discounted

sums of expectational terms. [For further details see Finn (1986).] Since $\Delta \log M_t$ and $\Delta \log P_t^*$ are covariance-stationary processes, the above results may be applied directly in equation (14'). This gives, after straightforward but lengthy rearrangement, the operationalized exchange rate equation, re-expressed in level form:

$$(21) \quad \log e_t = f_0 + d_1 \log M_t + d_2 \log M_{t-1} + d_3 \log M_{t-2} + d_4 \log P_t^* \\ + d_5 \log P_{t-1}^* + d_6 \log P_{t-2}^* + d_7 \log P_{t-3}^* + d_8 \log P_{t-4}^* \\ + d_9 \log c_t$$

where the new notation is:

$$d_1 \equiv h_1(1 + 1/\delta i_0) \quad d_2 \equiv h_1 \left[\frac{\alpha_{12}}{(1+\delta i_0)} - \frac{\alpha_{11}}{\delta i_0} \right] \\ d_3 \equiv - h_1 \frac{\alpha_{12}}{\delta i_0} \quad d_4 \equiv h_2 \left[\frac{\alpha_{21}}{(1+\delta i_0)} - 1 \right] \\ d_5 \equiv h_2 \left[\left(\frac{1}{1+\delta i_0} \right)^2 (\alpha_{21} \alpha_{22} - \alpha_{23}) + \left(\frac{1}{1+\delta i_0} \right)^3 \alpha_{23} \alpha_{21} \right] \\ d_6 \equiv h_2 \left[\left(\frac{1}{1+\delta i_0} \right)^2 (\alpha_{21} \alpha_{23} + \alpha_{22}^2) + \left(\frac{1}{1+\delta i_0} \right)^3 \alpha_{22} \alpha_{23} \right] \\ d_7 \equiv h_2 \left[\left(\frac{1}{1+\delta i_0} \right)^2 2\alpha_{22} \alpha_{23} + \left(\frac{1}{1+\delta i_0} \right)^3 \alpha_{23}^2 \right] \\ d_8 \equiv h_2 \left(\frac{1}{1+\delta i_0} \right)^2 \alpha_{23}^2 \quad d_9 \equiv - \sigma/\delta \\ h_1 \equiv \frac{\delta i_0}{(1+\delta i_0)} \left[\alpha_1 \left(\frac{1}{1+\delta i_0} \right) \right]^{-1} \\ h_2 \equiv \left[\alpha_2 \left(\frac{1}{1+\delta i_0} \right) \right]^{-1}$$

II.D. Estimation Technique and Diagnostics

The complete, operationalized model consists of equations (15), (16), (17) and (21). The first three of these are autoregressions, while (21) defines the

equilibrium relationship between all of the model's variables i.e. (21) is the "cointegrating regression" in the context of the present model. If (21) is correctly specified then the (omitted) disturbance term should be integrated of order zero, and since each of the included variables are integrated of order one, it follows that the vector consisting of these variables--which may be denoted by x_t --should be cointegrated of order (1, 1). In Engel and Granger's words: "Putting this another way, it means that equilibrium will occasionally occur, at least to a close approximation, whereas if x_t was not cointegrated, then . . . suggesting that in this case the equilibrium concept has no practical implications" [Engel and Granger (1987), p. 253]. More specifically, evidence suggesting that x_t is not cointegrated would in turn suggest that the particular equilibrium relationship in (21) is misspecified--either in terms of functional form or the omission of integrated-of-order-one variables.

Typically in the case of "cointegrating regressions" the equation disturbance term exhibits serial correlation. A Lagrange-Multiplier test on the consistent OLS residuals of equation (21) confirms the presence of serial correlation; in particular, the null hypothesis of first-order autocorrelation cannot be rejected.¹¹ Of course, if x_t is cointegrated, serial correlation must not be so strong as to imply a nonstationary disturbance term. The augmented Dickey-Fuller test was undertaken in this regard and the results suggested rejection of the null hypothesis of a nonstationary disturbance term.¹²

Assume, then, a first-order autoregressive process on the (omitted) disturbance term of equation (21), u_{et} , viz.:

$$(22) \quad u_{et} = \rho u_{et-1} + \epsilon_t$$

where: ϵ is a white noise disturbance term and ρ is a parameter, with $|\rho| < 1$ required to ensure the stationarity of u_{et} . The Cochrane-Orcutt transformation of (21) gives:

$$(23) \quad \log e_t = f_0(1-\rho) + \rho \log e_{t-1} + d_1(\log M_t - \rho \log M_{t-1}) + \\ d_2(\log M_{t-1} - \rho \log M_{t-2}) + d_3(\log M_{t-2} - \rho \log M_{t-3}) + \\ d_4(\log P_t^* - \rho \log P_{t-1}^*) + d_5(\log P_{t-1}^* - \rho \log P_{t-2}^*) + \\ d_6(\log P_{t-2}^* - \rho \log P_{t-3}^*) + d_7(\log P_{t-3}^* - \rho \log P_{t-4}^*) + \\ d_8(\log P_{t-4}^* - \rho \log P_{t-5}^*) + d_9(\log c_t - \rho \log c_{t-1}) + \epsilon_t$$

It is interesting to notice that equation (23) may be rearranged to give the "error-correction representation":

$$(23') \quad \Delta \log e_t = d_1 \Delta \log M_t + d_2 \Delta \log M_{t-1} + d_3 \Delta \log M_{t-2} \\ d_4 \Delta \log P_t^* + d_5 \Delta \log P_{t-1}^* + d_6 \Delta \log P_{t-2}^* + \\ d_7 \Delta \log P_{t-3}^* + d_8 \Delta \log P_{t-4}^* + d_9 \Delta \log c_t - \\ (1-\rho)u_{et-1} + \epsilon_t$$

where:

$$u_{et-1} = \log e_{t-1} - f_0 - d_1 \log M_{t-1} - d_2 \log M_{t-2} - d_3 \log M_{t-3} \\ - d_4 \log P_{t-1}^* - d_5 \log P_{t-2}^* - d_6 \log P_{t-3}^* - d_7 \log P_{t-4}^* \\ - d_8 \log P_{t-5}^* - d_9 \log c_{t-1}$$

i.e., equation (23') is exactly the error-correction representation of the model on using equations (15)-(17) to substitute for $\log M_t$, $\log P_t^*$ and $\log c_t$. The complete estimation system thus comprises equations (23) and (15)-(17), and the latter three equations are restated here for convenience:¹³

$$(15') \quad \log c_t = (1/\sigma) \log[\beta(1+r^*)] + \log c_{t-1}$$

$$(16) \quad \alpha_1(L) \Delta \log M_t = v_{1t}, \quad \alpha_1(L) \equiv 1 - \alpha_{11}L - \alpha_{12}L^2$$

$$(17) \quad \alpha_2(L) \Delta \log P_t^* = v_{2t}, \quad \alpha_2(L) \equiv 1 - \alpha_{21}L - \alpha_{22}L^2 - \alpha_{23}L^3$$

A diagnostic check on the Cochrane-Orcutt and OLS estimated residuals from equations (23) and (15'), respectively, was undertaken for possible problems such as outliers, functional misspecification and parameter instability, heteroscedasticity and non-normality--and also autocorrelation for the latter residuals.¹⁴ The results suggested that none of the aforementioned problems were present. Accordingly, estimation of the above joint system using the full-information-maximum-likelihood (FIML) technique is expected to yield consistent and asymptotically-efficient coefficient estimates.

All of the coefficients are identified with the exception of β and r^* in equation (15')--only the product, $\beta(1 + r^*)$, is identified. The highly nonlinear within- and cross-equation coefficient restrictions are tested by obtaining both the restricted and unrestricted FIML estimates and carrying out a likelihood ratio test. The restricted system is the estimation system specified above with the definitions of the coefficients, d_1 - d_9 , explicitly imposed. The unrestricted system differs by not imposing these definitions and by treating the constant term in (15') (denoted d_{10} below) as a free parameter. There are, therefore, seven restrictions.

III. The Empirical Findings

The FIML estimates of the restricted and unrestricted systems are presented in Table 2.^{15,16} The associated likelihood ratio test statistic is 5.86, which when compared to the χ^2 statistic at seven degrees of freedom: 14.07, indicates that the coefficient restrictions, discussed above, cannot be rejected at a 5% level of significance.

The point estimates of the preference parameters, namely $\sigma = 3.13$ and $\delta = 0.67$, are in the concave region of parameter space and, based on a one-tailed t test, are significantly greater than zero at a 10% level of significance--marginally falling short of significance at the 5% level. In addition, a two-standard-deviation confidence interval for each of these estimates includes unity--a parameter value associated with a logarithmic preference specification in respect to consumption and real money balances. The point estimate of σ compares to other estimates of this parameter, obtained in other studies. In particular, Hansen and Singleton (1982, 1983, 1984) estimated and tested a barter-economy model of US stock returns and found values of σ , typically, ranging from zero to two. Finn, Hoffman and Schlagenhauf (1988) estimated and tested a monetary-economy model of US stock returns and found values of σ ranging between 0.02 and 4.8.¹⁷ The point estimate of $\beta(1 + r^*)$ is 1.01, very precise, and significantly greater than zero. This estimate suggests that the real interest rate is greater than the subjective rate of time preference--but notice that a two-standard-deviation confidence interval also includes unity and values slightly less than unity, so the results are also consistent with equality or the opposite relationship between these two parameters. The point estimate of ρ is 0.96, it is significantly greater than zero and a two-standard deviation confidence interval contains unity. Accordingly, while this point estimate is consistent with the stationarity of the disturbance term in the exchange rate equation, and thus supportive of the specified equilibrium relationship among the model's variables, its high value and insignificance from unity may point to misspecification of that relationship. The estimated coefficients of the time-series processes for the domestic money supply and foreign price level

closely correspond in both magnitude and significance to the Box-Jenkins estimates reported in Table 1.

Next, consider the explanatory power of the restricted model: The (top) right-most columns of Table 2 list the sum-of-squared residuals, the standard error of the regression and the Durbin-Watson statistic for each of the system's equations. The former two statistics suggest that each equation is well fitted. A Durbin-Watson (Durbin-h) test for the consumption (exchange rate) equation points to rejection of the null hypothesis of serial correlation. A Lagrange-Multiplier test on the residuals of the money supply and price level equations also rejects the null hypothesis of serial correlation for these equations.¹⁸ In view of the interest in the literature, referred to above, in comparing the tracking/forecasting performance of structural economic models versus a random walk model of the exchange rate, one such comparison is also undertaken here. In particular, the restricted-system reduced-form solution for the exchange rate --using the parameter estimates given in Table 2--is used to 'forecast' the one-period ahead exchange rate over period 1974:2 to 1986:4.¹⁹ The relative forecasting performance of this model versus the random walk model is summarized in terms of the mean-absolute and root-mean-squared error statistics reported in Table 3. As indicated there, the 'forecasting' performance of both models is quite good and similar, with the random-walk model having a slight edge over the structural model of the present study. This result may appear somewhat disappointing on first sight--especially in view of the fact that the structural model embodies more information than does the random walk model (since the former uses estimates based on the entire sample period). However, on the contrary, it may be argued that this forecasting performance of the structural model is remarkably good in view of the fact that the model is estimated using a maximum-

likelihood--which does not minimize the sum-of-squared residuals of each equation.

IV. Summary and Conclusion

This paper attempts the econometric implementation of a monetary intertemporal general-equilibrium model of exchange rate and current account determination in a small open economy. In this model agents are viewed as facing perfect goods and asset markets and choosing consumption and savings plans so as to maximize the utility from consumption and real money balances over an infinite time horizon, subject to an intertemporal budget constraint. The hallmark of such a model is that current and expected future real opportunities and monetary policies play a key role in the money demand decision that governs the evolution of the exchange rate, while current and expected future real opportunities influence the consumption-savings decision that determines the current account. The reduced form of the model essentially comprises a statement of the permanent income hypothesis of consumption and the rational expectations extension of the monetary approach to exchange rate determination. The model is operationalized using the results of Hansen and Sargent (1980) and Flavin (1981) for operationalizing infinite-discounted sums of expectational terms and Box-Jenkins time-series techniques to characterize the exogenous driving processes. The model is then estimated using full-information-maximum-likelihood methods and quarterly data over the recent flexible exchange rate period on the UK and US economies, where the UK is taken as the small open economy.

The empirical findings suggest that the intertemporal general-equilibrium framework shows considerable promise in our ability to explain the exchange rate and consumption side of current account behavior. More specifically, the highly

nonlinear within-and cross-equation coefficient restrictions specified by the model are not rejected; estimates of the 'deep structural' parameters of taste and the exogenous driving processes are plausible in magnitude and are either highly significant or close to significance at reasonable levels of confidence. In addition, the explanatory power of the model is quite good and the reduced-form 'forecasting' performance of the model, with respect to the exchange rate, closely matches that of the random walk model. The finding of a highly-autocorrelated and almost nonstationary disturbance term in the exchange rate equation is, however, suggestive of the omission of an integrated-of-order-one variable and/or misspecification of the functional form of the model.

In view of the above findings, a number of challenging but exciting extensions of the intertemporal framework used in the present study seem extremely fruitful: First, since real exchange rate movements are both significant and highly persistent [see e.g., Huizinga (1987)], it would be interesting to extend the model to admit and explain real exchange rate variability--perhaps along the lines in Stockman (1983). Second, it may be useful to allow for time-varying real interest rates, given evidence in this regard [see e.g., Mishkin (1984)]. In a more general model with time-varying and--for a large open economy--endogenous real interest rates, government expenditures would have real interest rate effects as modeled in Barro (1981). Third, it may be interesting to relax the assumption of Ricardian Equivalence--admitting the possibility of real effects from alternative financing schemes for government expenditures involving taxation and bond issue. Fourth, the model may be extended to allow the marginal utility of consumption depend on real money balances, thus allowing for possible monetary non-neutralities and non-superneutralities. This extension of the model seems especially fruitful in view

of the evidence in Finn, Hoffman, and Schlagenhauf (1988) that models allowing for such non-neutralities are more consistent with the data on US stock returns. Finally, it would be interesting to extend the model to include an explanation of the investment side of current account determination--perhaps along the time-to-build lines of Kydland and Prescott (1982).

Appendix 1

Data and Data Sources

- e: UK pound per US dollar; end of period; source 1.
- M: UK M1; end of period; seasonally adjusted; source 1.
- P*: US GNP Price Deflator; 1980 = 100; seasonally adjusted; source 1.
- c: UK consumer expenditure on nondurable goods and services at 1980 prices; seasonally adjusted; source 2.

Source 1: OECD Main Economic Indicators: Historical Statistics, various issues.

Source 2: UK Central Statistics Office: Economic Trends Annual Supplement, 1988 edition.

Appendix 2

This Appendix provides the details on the derivation of equation (14) in the main text.

First, set $M_t^d = M_t$ and substitute for c_{t+1} in (4), using (3), to get:

$$(A1) \quad \left(\frac{M_t}{P_t}\right)^{\delta} = c_t^{-\sigma} \left[1 - \left(\frac{P_{t+1}}{P_t}\right)^{-1} (1+r^*)^{-1}\right]$$

Notice from (A1) that σ , δ determine the elasticity of intratemporal substitution between consumption and real money balances. Taking (natural) logarithms of (A1) gives:

$$(A2) \quad -\delta \log M_t + \delta \log P_t = -\sigma \log c_t + \log \left[1 - \left(\frac{P_{t+1}}{P_t}\right)^{-1} (1+r^*)^{-1}\right]$$

Next note $\frac{P_{t+1}}{P_t} (1+r^*) \equiv (1+i_t)$ and the following approximations:

$$\log \left[1 - (1+i_t)^{-1}\right] \approx \log i_t \approx \log i_0 + 1/i_0 [i_t - i_0]$$

The second approximation involves a first-order Taylor-series expansion of the $\log i_t$ around the mean of i_t , i_0 . Using these approximations, and equations (11)-(13) in (A2) and rearranging gives:

$$(A3) \quad \log e_t = \frac{\delta i_0}{1 + \delta i_0} \log M_t - \log P_t^* + \frac{1}{1 + \delta i_0} \log P_{t+1}^* \\ - \frac{\sigma i_0}{1 + \delta i_0} \log c_t + \frac{1}{1 + \delta i_0} \log e_{t+1} + s_0 \\ \text{where } s_0 = \frac{i_0}{1 + \delta i_0} (\log i_0 + r^*/i_0 - 1)$$

This is a first order difference equation in $\log e_t$. Since the coefficient on $\log e_{t+1}$ is a (positive) fraction, solving (A3) "forward" will ensure that the solution maps the bounded sequences of M_t , P_t^* and c_t into a bounded sequence for e_t . Solving (A3) forward, noting (9), and rearranging gives equation (14) in the text.

References

- Backus, D., "Empirical Models of the Exchange Rate: Separating the Wheat from the Chaff," Canadian Journal of Economics, November 1984, 17: 824-846.
- Barro, R., "Output Effects of Government Purchases," Journal of Political Economy, December 1981, 89: 1086-1119.
- Bilson, J., "Rational Expectations and the Exchange Rate," in J. Frenkel and H. Johnson, eds., The Economics of Exchange Rates, Reading, Mass.: Addison-Wesley Press, 1978.
- Brock, P., "Investment, The Current Account and The Relative Price of Non-Traded Goods in a Small Open Economy," Journal of International Economics, May 1988, 24: 235-253.
- Chow, G., Econometrics, New York, McGraw-Hill, 1983.
- Cumby, R., and M. Obstfeld, "International Interest-Rate and Price-Level Linkages Under Flexible Exchange Rates: A Review of Recent Evidence," NBER WP No. 921, June 1982.
- Engel, R., and C. Granger, "Co-integration and Error Correction: Representation, Estimation and Testing", Econometrica, March 1987, 55: 251-276.
- Feenstra, R., "Functional Equivalence Between Liquidity Costs and the Utility of Money," Journal of Monetary Economics, March 1986, 17: 271-292.
- Finn, M., "Forecasting the Exchange Rate: A Monetary or Random Walk Phenomenon?," Journal of International Money and Finance, June 1986, 5: 181-193.
- Finn, M., D. Hoffman and D. Schlagenhauf: "Intertemporal Asset-Pricing Relationships In Barter and Monetary Economies: An Empirical Analysis," University of Western Ontario Research Report No. 8805, June 1988.
- Flavin, M., "The Adjustment of Consumption to Changing Expectations about Future Income," Journal of Political Economy, October 1981, 89: 974-1009.

- Frenkel, J., "The Collapse of Purchasing Power Parities During the 1970's," European Economic Review, May 1981, 16: 145-165.
- Greenwood, J., "Expectations, the Exchange Rate and the Current Account," Journal of Monetary Economics, November 1983, 12: 543-569.
- Hall, R.E., "Intertemporal Substitution In Consumption," Journal of Political Economy, April 1988, 96: 339-357.
- Hansen, L., and T. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models," Journal of Economic Dynamics and Control, February 1980, 2: 7-66.
- Hansen, L., and K. Singleton, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," Econometrica, September 1982, 50: 1269-86.
- _____, "Stochastic Consumption, Risk Aversion and the Temporal Behavior of Asset Returns," Journal of Political Economy, April 1983, 91: 249-265.
- _____, "Errata," Econometrica, January 1984, 52: 267-268.
- Hayashi, F., "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables," Journal of Political Economy, October 1982, 90: 895-916.
- Helpman, E., and A. Razin, "Dynamics of a Floating Exchange Rate Regime" Journal of Political Economy, August 1982, 90: 728-754.
- Hercowitz, Z., "On the Determination of the External Debt: The Case of Israel", Journal of International Money and Finance, September 1986, 5: 315-334.
- Hodrick, R., "Risk, Uncertainty and Exchange Rates", NBER WP No. 2429, 1987.
- Huizinga, J., "An Empirical Investigation of the Long Run Behavior of Real Exchange Rates", in K. Brunner and A. Meltzer, eds., Carnegie-Rochester Conference Series on Public Policy, Autumn 1987, 27: 149-214.

- Jarque, C., and A. Bera, "Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals," Economic Letters, No. 3, 1980, 6: 255-259.
- Kydland, F., and E. Prescott, "Time to Build and Aggregate Fluctuations", Econometrica, November 1982, 50: 1345-1370.
- Lipton, D., and J. Sachs, "Accumulation and Growth in a Two-Country Model: A Simulation Approach," Journal of International Economics, August 1983, 15: 135-159.
- Lucas, R.E., "Econometric Policy Evaluation: A Critique," in K. Brunner and A. Meltzer, eds., The Phillips Curve and Labor Markets, Amsterdam: North-Holland, 1976.
- Lucas, R. E., "Interest Rates and Currency Prices in a Two-Country World", Journal of Monetary Economics, November 1982, 10: 335-360.
- _____, "Money In a Theory of Finance", in K. Brunner and A. Meltzer, eds., Carnegie-Rochester Conference Series on Public Policy, Autumn 1984, 21: 9-46.
- Meese, R., and K. Rogoff, "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?," Journal of International Economics, February 1983, 14: 3-24.
- Mishkin, F., "The Real Interest Rate: A Multi-Country Empirical Study," Canadian Journal of Economics, May 1984, 17: 283-311.
- Obstfeld, M., "Macroeconomic Policy, Exchange-Rate Dynamics, and Optimal Asset Accumulation", Journal of Political Economy, December 1981, 89: 1142-1161.
- _____, "Capital Mobility in the World Economy: Theories and Measurement," in K. Brunner and A. Meltzer, eds., Carnegie-Rochester Conference Series on Public Policy, Spring 1986, 24: 55-104.

- Pagan, A., and A. Hall, "Diagnostic Tests as Residual Analysis," Australian National University Working Paper No. 087, April 1983.
- Pindyck, R., and D. Rubinfeld, Econometric Models and Economic Forecasts, New York: McGraw-Hill Book Co., 1976.
- Prescott, E., "Theory Ahead of Business-Cycle Measurement," Federal Reserve Bank of Minneapolis Quarterly Review, Fall 1986, 10:9-22.
- Sachs, J., "The Current Account and Macroeconomic Adjustment in the 1970s" Brookings Paper on Economic Activity, 1981, 1: 201-282.
- Stockman, A., "Real Exchange Rates Under Alternative Nominal Exchange Rate Systems", Journal of International Money and Finance, August 1983, 2: 147-166.
- Stockman, A., and L. Svensson, "Capital Flows, Investment and Exchange Rates", Journal of Monetary Economics, March 1987, 19: 171-202.
- Svensson, L., "Money and Asset Prices in a Cash-In-Advance Economy", Journal of Political Economy, October 1985, 93: 919-944.
- Svensson, L., and A. Razin, "The Terms of Trade and the Current Account: The Harberger-Laursen-Metzler Effect", Journal of Political Economy, February 1983, 91: 97-125.
- White, M., "A Heteroskedasticity-Consistent Covariance Matrix Estimator and A Direct Test for Heteroskedasticity," Econometrica, May 1980, 48: 817-838.

NOTES

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1. The real investment decision would also enter the determination of the current account in a more general model.
 2. It is also noteworthy that the generalized-method-of-moments estimation of the Euler equations derived from the stochastic model cannot be justified in this study since no stationarity-inducing transformation of the Euler equation governing money holdings is possible.
 3. Hodrick's (1987) study is based on Svensson's (1985) model but in solving the model it is assumed that the parameters are such that the CIA constraint is always binding and hence a unitary income velocity results.
 4. The momentary utility function in (1) is assumed to be separable across consumption and real money balances - this greatly simplifies the solution to the model. The implication is that the marginal utilities of consumption and real money balances are independent of one another. An isoelastic functional form is chosen for each component of the momentary utility function in view of its prevalence in the macroeconomic asset-pricing [see e.g., Hansen and Singleton (1983)] and real business cycle [Prescott (1986)] literatures. This formulation implies both a constant

elasticity of intertemporal substitution of consumption and a constant elasticity of intratemporal substitution between consumption and real money balances.

5. In a stochastic environment σ is the coefficient of relative risk aversion. See Hall (1988) for an interesting discussion with respect to interpreting $1/\sigma$ as the elasticity of intertemporal substitution while not revealing anything about risk aversion in a stochastic environment with more general specifications of utility than that given in equation (1).
6. In deriving this result it is assumed that: $(1 + r^*)^{-1} [\beta(1 + r^*)]^{1/\sigma} < 1$. This condition holds unambiguously for $\sigma > 1$.
7. In order to see this point more clearly note that a typical expression of the 'simple' monetary approach is:

$$(*) \log e_t = \log M_t - \log P_t^* - \eta \log y_t^m + \epsilon_i t + a_0$$

where: y_t^m denotes a real income measure (current/permanent), $\eta(\epsilon)$ denotes the real-income (interest rate semi-) elasticity of the demand for money and a_0 is a constant term [see e.g., Bilson (1978) and Finn (1986)]. On the other hand, with regard to the model used in the present study: equation (A2), the approximations noted in Appendix 2 together with equations (10) and (12) in the text and the definition of permanent income may be used to give:

$$(**) \log e_t = \log M_t - \log P_t^* - \sigma/\delta \log y_t^p + (1/\delta i_0) i_t + d_0$$

$$\text{where: } d_0 = 1/\delta [\log i_0 - 1] - \sigma/\delta \log \left[\frac{m(1+r^*)}{r^*} \right]$$

Clearly, equations (*) and (**) compare closely. The 'rational expectations' versions of each of these equations is, essentially, obtained by substituting for i_t , using the uncovered interest rate parity condition,

equation (11) in the text, and solving the resulting equation forward for $\log e_t$.

8. Notice that it is possible to operationalize the consumption function in equation (10) by exploiting the first-order difference equation for the human wealth component of total wealth, as in Hayashi (1982). This may be outlined as follows. Define the human wealth component of W_t as:

$$H_t \equiv \sum_{s=0}^{\infty} \frac{(y_{t+s} - g_{t+s})}{(1 + r^*)^s}$$

the evolution of which (following by definition) is governed by:

$$H_t = (1 + r^*)(H_{t-1} - y_{t-1} + g_{t-1})$$

Use equation (10) and the one-period lagged version thereof to substitute, respectively, for H_t and H_{t-1} in the foregoing equation and note equation (6) to obtain:

$$c_t = [\beta(1 + r^*)]^{1/\sigma} c_{t-1}$$

i.e. equation (15). This shows that equation (15) is consistent with the budget constraint of the maximization problem.

9. Note these variables were entered in first difference form as first differencing was necessary to achieve stationarity.
10. Since the stochastic processes for the exogenous variables are modeled as autoregressive processes, all current period shocks decay through time at a speed determined by the magnitude of the autoregressive parameters. The larger are the latter, the more long-lasting are the effects of current shocks.
11. The null hypothesis of higher-order autocorrelation was rejected. See Pagan and Hall (1983) for the details of the Lagrange Multiplier test.

12. Engel and Granger (1987) provide the details of the augmented Dickey-Fuller test. In view of the quarterly nature of the data used in our study, four lagged values of the first-differenced disturbances from the "cointegrating regression" were employed. Engel and Granger tabulate the critical values for the t-statistic involved in this test for one sample size (100 observations) and only for the bivariate case. Accordingly, conducting a test in our case, using their critical values, is strictly speaking only suggestive. A t-statistic equal to -3.35 compared to the 5% critical value, -3.17, suggests rejection of the null hypothesis of a nonstationary error term.
13. Notice that the consumption function is expressed in logarithmic form in the estimation system. This is so because consumption enters the exchange rate equation in logarithmic form. It is important that a variable be measured in the same units consistently throughout a nonlinear model, which has shared parameters across equations, in order to ensure meaningful parameter estimates.
14. The procedures employed are briefly indicated here:
 - (a) A time-series plot of the residuals was examined for outlying residuals.
 - (b) A Chow test for functional misspecification and parameter instability was conducted. See Chow (1983) for details.
 - (c) The White (1980) test was used to check for heteroscedasticity. Note that only an approximation to the White test was undertaken on the residuals from the exchange rate equation since the large number of all bivariate combinations of the explanatory variables in that equation would, if included in the test regression, exhaust the degrees

of freedom. Here only all bivariate combinations of the contemporaneous explanatory variables were included. A Bartlett test for heteroscedasticity was, thus, also undertaken for the exchange rate equation [see Pindyck and Rubinfeld (1976) for details].

(d) The Jarque-Bera (1980) test for the non-normality of the residuals was used.

(e) A Lagrange-Multiplier (LM) test was used in checking for autocorrelation [see Pagan and Hall (1983) for details].

15. The estimates and log of the likelihood function were robust to alternative starting values. This suggests that the likelihood function is well-behaved and that a global maximum has been achieved. The starting values used for the results reported in Table 2 are:

(a) for the restricted system: the Box-Jenkins estimates of the time-series parameters, $f_0 = 5$, $\sigma = \delta = 1$ and $\beta(1+r^*) = \rho = 0.95$;

(b) for the unrestricted system: the Box-Jenkins estimates of the time-series parameters and, for the remaining parameters, the solution values implied by the restricted estimates in Table 2.

16. An unrestricted system differing from that described above by treating the coefficient on each individual right-hand-side variable entering into equation (23) as a free parameter (thirteen in total) was also estimated and the likelihood ratio test suggested that the associated coefficient restrictions (ten in total) also could not be rejected at a 5% level of significance.

17. I do not know of any other study which presents an estimate of the δ parameter.

18. The Durbin-h test breaks down for these two equations as the calculated h-statistic is negative (the test is valid only for positive h-statistic values). The Durbin-h rather than Durbin-Watson test is used in checking for first-order autocorrelation whenever the equation of interest includes the estimated effect of a lagged dependent variable.
19. This is not, strictly speaking, a forecasting exercise since the coefficient estimates embody information over the entire sample period. The exercise is, nonetheless, illustrative of the model's ability to track the data.

Table 1: Box - Jenkins Estimation and Diagnostic Test Results
Sample Period: 1974:2 to 1986:4

Equation for:	$\Delta \log M_t$		$\Delta \log P_t^*$		
coefficients (t values)	$\alpha_{11} = 0.35$ (2.60)	$\alpha_{12} = 0.56$ (4.02)	$\alpha_{21} = 0.43$ (3.25)	$\alpha_{22} = 0.14$ (0.93)	$\alpha_{23} = 0.43$ (3.31)
Autocorrelations of residuals	<u>1 - 10</u> - 0.10 - 0.22 - 0.06 0.04 - 0.09 0.04 - 0.22 - 0.03 0.23 0.02	<u>11 - 20</u> - 0.20 0.04 0.16 - 0.10 0.13 - 0.09 0.02 0.03 0.08 - 0.23	<u>1 - 10</u> 0.03 0.04 0.16 0.04 0.07 - 0.17 - 0.17 - 0.11 0.12 - 0.11	<u>11 - 20</u> - 0.19 0.13 - 0.15 0.06 - 0.03 - 0.12 - 0.04 - 0.19 - 0.13 0.04	
Box - Pierce Q Statistic	22.40		18.89		
Critical Value of χ^2_{m-q}	28.87		27.59		

- (i) Source of data: OECD Main Economic Indicators
- (ii) The Q statistic reported is for 20 autocorrelations of the residuals.
- (iii) χ^2_{m-q} is the chi-squared statistic with (m-q) degrees of freedom, where
m = number of autocorrelations and q = number of parameter estimates.

Table 2: FIML Estimates of the Restricted and Unrestricted Systems
Sample Period: 1974:2 to 1986:4

<u>Restricted System</u>		<u>Log of Likelihood</u> = 587.57		<u>Equation Statistics</u>				
					<u>SSR</u>	<u>SER</u>	<u>DW</u>	
σ	= 3.13 (1.47)	δ	= 0.67 (1.66)	(i)	Log e_t eq.	0.52	0.10	2.02
$\beta(1+r^*)$	= 1.01 (114.03)	ρ	= 0.96 (29.74)	(ii)	Log c_t eq.	0.004	0.01	1.82
f_0	= 9.87 (7.76)	α_{11}	= 0.35 (2.26)	(iii)	Δ Log M_t eq.	0.02	0.02	1.77
α_{12}	= 0.49 (2.85)	α_{21}	= 0.43 (2.18)	(iv)	Δ Log P_{t-1}^* eq.	0.001	0.004	1.79
α_{22}	= 0.16 (0.72)	α_{23}	= 0.37 (2.54)					

<u>Unrestricted System</u>		<u>Log of Likelihood</u> = 590.50		<u>Equation Statistics</u>				
					<u>SSR</u>	<u>SER</u>	<u>DW</u>	
ρ	= 0.986 (53.59)	d_1	= 3.26 (7.75)	(i)	Log e_t eq.	0.43	0.09	1.94
d_2	= -0.98 (1.25)	d_3	= -2.24 (2.75)	(ii)	Log c_t eq.	0.004	0.008	1.82
d_4	= -5.63 (2.72)	d_5	= -0.90 (0.35)	(iii)	Δ Log M_t eq.	0.02	0.02	1.81
d_6	= 2.37 (1.20)	d_7	= 3.85 (1.53)	(iv)	Δ Log P_t^* eq.	0.001	0.004	1.79
d_8	= -0.07 (0.03)	d_9	= -5.08 (4.61)					
d_{10}	= 0.005 (2.86)	α_{11}	= 0.36 (1.98)					
α_{12}	= 0.55 (2.85)	α_{21}	= 0.44 (2.11)					
α_{22}	= 0.14 (0.59)	α_{23}	= 0.39 (2.15)					

- (i) Source of data: OECD Main Economic Indicators (various issues),
UK Central Statistics Office: Economic Trends Annual Supplement, 1988.
- (ii) t statistics are reported in parenthesis
- (iii) SSR denotes the sum-of-squared residuals
SER denotes the standard error of the regression
DW denotes the Durbin-Watson statistic.

Table 3: Forecasting Performance Summary Statistics

	<u>MODEL</u>	<u>RW</u>
RMSE	0.05880.0522	
MAE	0.04790.0419	

- (i) RMSE, MAE denote the root-mean-squared and mean-absolute error.
- (ii) MODEL, RW denote the reduced-form restricted model of the present study and the random-walk model.